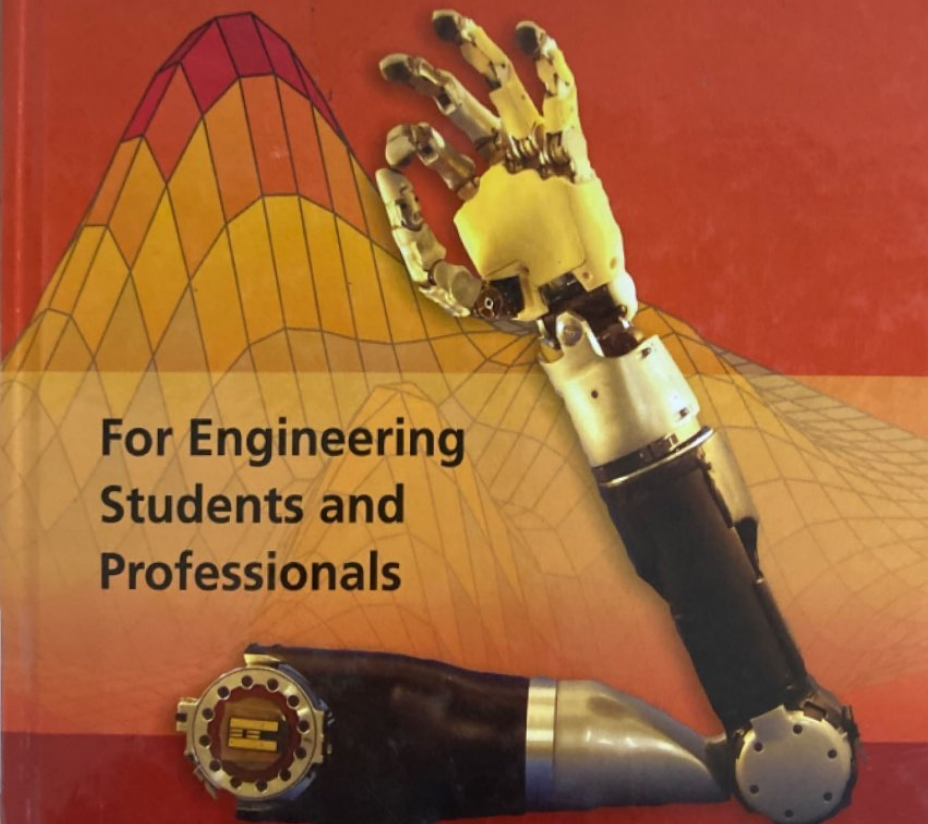


Optimization in Practice with **MATLAB®**

A detailed image of a yellow and black robotic hand and forearm. The hand is positioned as if resting on a wireframe mountain. The mountain is a 3D grid of yellow and orange polygons, with the peak colored in red. The background is a solid red color.

**For Engineering
Students and
Professionals**

ACHILLE MESSAC

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