TEXTBOOKS IN MATHEMATICS

Numerical Analysis for Engineers

Methods and Applications

SECOND EDITION

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