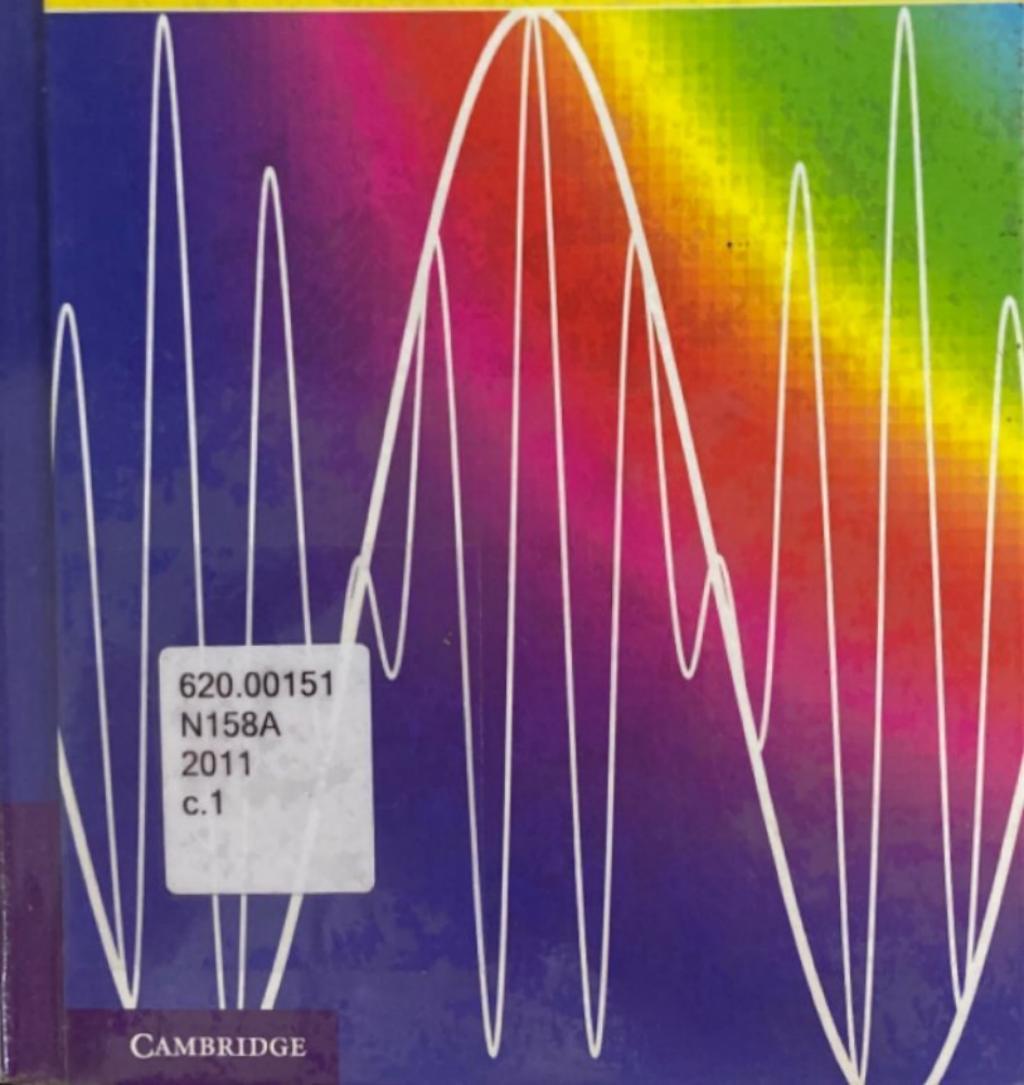


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ADVANCED TOPICS IN APPLIED MATHEMATICS

For Engineering and the Physical Sciences



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